ISSN: **1304-7981** Number: **10**, Year:**2016**, Pages: **10-24** **http://jnrs.gop.edu.tr**

Gaziosmanpasa University
Graduate School of Natural and Applied Sciences
Journal of New Results in Science

Received: 06.08.2014
Accepted: 24.02.2016

Editors-in-Chief: *Bilge Hilal Çadırcı*
Area Editor: *Serkan Demiriz*

On New Decompositions of Some Soft Continuity Types

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Abstract - In this paper, we introduce the concepts of δ - open soft sets, β^* - soft sets in a soft topological space and so we define the notions of δ - soft continuity, β^* - soft continuity between soft topological spaces via δ - open, β^* - soft sets. Also, we obtain decompositions of semi - soft continuity and α - soft continuity via δ - soft continuity. Finally, we show that a function is soft continuous if and only if it is both α - soft continuous and β^* - soft continuous.

Keywords - *δ - open soft set, δ - soft continuity, β^* - soft set, β^* - soft continuity.*

1 Introduction

The notion of soft sets was first introduced by Molodtsov [5] in 1999 as a general mathematical tool to cope with unclear concepts. He has shown that very important applications of soft set theory such as dealing with some problems in medical science, economics etc. There has been an extensive study on the importance, properties and applications of soft set theory [1, 2, 4, 6, 7]. The notion of continuity is an important concept in general topology, fuzzy topology, generalized topology etc. as well as in all branches of mathematics. In recent years, Zorlutuna et al. [7] defined the image (resp.

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inverse image) of a soft set under a function and soft continuity. Also, Kandil et al. [3], defined semi - open (resp. pre - open, α - open, β - open) soft set and semi (resp. pre, α, β) - soft continuity via these soft sets.

The purpose of the present paper is to introduce the concepts of δ - open soft sets, β^* - soft sets in a soft topological space and so we define the notions of δ - soft continuity, β^* - soft continuity between soft topological spaces via δ - open, β^* - soft sets. In Section 3, we define δ - open soft sets, β^* - soft sets and obtain some properties, characterizations of these soft sets. In Section 4, we obtain decompositions of semi - soft continuity and α - soft continuity via δ - soft continuity. Finally, we show that a function is soft continuous if and only if it is both α - soft continuous and β^* - soft continuous.

2 Preliminary

In this section we recall some known definitions and theorems.

Definition 2.1. *[5] A pair (F, A), where F is mapping from A to $P(X)$, is called a soft set over X. The family of all soft sets on X denoted by $SS(X)_E$.*

Definition 2.2. *[4] Let (F, A) and (G, B) be two soft sets over a common universe X. Then (F, A) is said to be a soft subset of (G, B) if $A \subseteq B$ and $F(e) \subseteq G(e)$, for all $e \in A$. This relation is denoted by $(F, A)\widetilde{\subseteq}(G, B)$.*

(F, A) is said to be soft equal to (G, B) if $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$. This relation is denoted by $(F, A) = (G, B)$.

Definition 2.3. *[1] The complement of a soft set (F,A) is defined as $(F, A)^c = (F^c, A)$, where $F^c(e) = (F(e))^c = X - F(e)$, for all $e \in A$.*

Definition 2.4. *[6] The difference of two soft sets (F,A) and (G,A) is defined by $(F, A) - (G, A) = (F - G, A)$, where $(F - G)(e) = F(e) - G(e)$, for all $e \in A$.*

Definition 2.5. *[6] Let (F, A) be a soft set over X and $x \in X$. x is said to be in the soft set (F, A) denoted by $x \in (F, A)$ if $x \in F(e)$ for all $e \in A$.*

Definition 2.6. *[4] A soft set (F, A) over X is said to be a null soft set if $F(e) = \emptyset$, for all $e \in A$. This denoted by $\widetilde{\emptyset}$.*

Definition 2.7. *[4] A soft set (F, A) over X is said to be an absolute soft set if $F(e) = X$, for all $e \in A$. This denoted by $\widetilde{X}$.*

Definition 2.8. *[4] The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and $H(e) = F(e)$ if $e \in A - B$ or $H(e) = G(e)$ if $e \in B - A$ or $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$ for all $e \in C$.*

Definition 2.9. *[4] The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.*

Now we recall the definition of soft topological spaces as follows:

Definition 2.10. *[6] Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X if*

1. *$\widetilde{\emptyset}$, $\widetilde{X} \in \tau$;*
2. *the intersection of any two soft sets in τ belongs to τ;*
3. *the union of any number of soft sets in τ belongs to τ.*

The triple (X, τ, E) is called a soft topological space over X. The members of τ are said to be $\tau-$ soft open sets or soft open sets in X. A soft set over X is said to be soft closed in X if its complement belongs to τ. The set of all open soft sets over X denoted by $OS(X, \tau, E)$ or $OS(X)$ and the set of all closed soft sets denoted by $CS(X, \tau, E)$ or $CS(X)$.

Definition 2.11. *[6] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. The soft closure of (F, E), denoted by $cl(F, E)$ or $\overline{(F, E)}$ is the intersection of all closed soft super sets of (F, E).*

Definition 2.12. *[7] Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. The soft interior of (F, E), denoted by $int(F, E)$ or $(F, E)^o$ is the union of all open soft subsets of (F, E).*

The following theorems give some properties of soft closure and soft interior of a soft set.

Theorem 2.13. *[2] Let (X, τ, E) be a soft topological space over X, (F, E) and (G, E) are soft sets over X. Then*

1. *$cl\widetilde{\emptyset} = \widetilde{\emptyset}$ and $cl\widetilde{X} = \widetilde{X}$.*
2. *$(F, E)\widetilde{\subseteq}cl(F, E)$.*
3. *(F, E) is a closed set if and only if $(F, E) = cl(F, E)$.*
4. *$cl(cl(F, E)) = cl(F, E)$.*
5. *$(F, E)\widetilde{\subseteq}(G, E)$ implies $cl(F, E)\widetilde{\subseteq}cl(G, E)$.*
6. *$cl((F, E) \cup (G, E)) = cl(F, E) \cup cl(G, E)$.*

7. $cl((F,E)\cap(G,E))\widetilde{\subseteq}cl(F,E)\cap cl(G,E)$.

Theorem 2.14. *[2] Let (X,τ,E) be a soft topological space over X and (F,E) and (G,E) are soft sets over X. Then*

1. $int\widetilde{\emptyset}=\widetilde{\emptyset}$ *and* $int\widetilde{X}=\widetilde{X}$.
2. $int(F,E)\widetilde{\subseteq}(F,E)$.
3. $int(int(F,E))=int(F,E)$.
4. (F,E) *is a soft open set if and only if* $int(F,E)=(F,E)$.
5. $(F,E)\widetilde{\subseteq}(G,E)$ *implies* $int(F,E)\widetilde{\subseteq}int(G,E)$.
6. $int(F,E)\cap int(G,E)=int((F,E)\cap(G,E))$.
7. $int(F,E)\cup int(G,E)\widetilde{\subseteq}int((F,E)\cup(G,E))$.

Definition 2.15. *[3] Let (X,τ,E) be a soft topological space and $(F,E)\in SS(X)_E$. Then (F,E) is said to be*

1. *pre - open soft set if* $(F,E)\widetilde{\subseteq}int(cl(F,E))$;
2. *semi - open soft set if* $(F,E)\widetilde{\subseteq}cl(int(F,E))$;
3. α *- open soft set if* $(F,E)\widetilde{\subseteq}int(cl(int(F,E)))$;
4. β *- open soft set if* $(F,E)\widetilde{\subseteq}cl(int(cl(F,E)))$.

The set of all pre - open soft sets (resp. semi - open soft sets, α - open soft sets, β - open soft sets) denoted by $POS(X)$ (resp. $SOS(X)$, $\alpha OS(X)$, $\beta OS(X)$).

The following definitions introduce the image (resp. inverse image) of soft set under the funtion and some soft continuities of functions.

Definition 2.16. *[7] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets, $u:X\to Y$ and $p:A\to B$ be mappings. Then the function $f_{pu}:SS(X)_A\to SS(Y)_B$ is defined as:*

1. *Let $(F,A)\in SS(X)_A$. The image of (F,A) under f_{pu}, written as $f_{pu}(F,A)=(f_{pu}(F),p(A))$, is a soft set in $SS(Y)_B$ such that $f_{pu}(F)(y)=\bigcup_{x\in p^{-1}(y)\cap A}u(F(x))$ if $p^{-1}(y)\cap A\neq\emptyset$ and $f_{pu}(F)(y)=\emptyset$ if $p^{-1}(y)\cap A=\emptyset$ for all $y\in B$.*
2. *Let $(G,B)\in SS(Y)_B$. The inverse image of (G,B) under f_{pu}, written as $f_{pu}^{-1}(G,B)=(f_{pu}^{-1}(G),p^{-1}(B))$, is a soft set in $SS(X)_A$ such that $f_{pu}^{-1}(G)(x)=u^{-1}(G(p(x)))$ if $p(x)\in B$ and $f_{pu}^{-1}(G)(x)=\emptyset$ if $p(x)\notin B$ for all $x\in A$.*

Definition 2.17. *[7] Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces and f_{pu} : $SS(X)_A \to SS(Y)_B$ be a function. Then*

1. *The function f_{pu} is called soft continuous (soft - cts) if $f_{pu}^{-1}(G, B) \in \tau$ for all $(G, B) \in \tau^*$.*

2. *The function f_{pu} is called an open soft function if $f_{pu}(G, A) \in \tau^*$ for all $(G, A) \in \tau$.*

Definition 2.18. *[3] Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then*

1. *The function f_{pu} is called a pre - soft continuous function (briefly, Pre - cts soft) if $f_{pu}^{-1}(G, B) \in POS(X)$ for all $(G, B) \in OS(Y)$.*

2. *The function f_{pu} is called a α - soft continuous function (briefly, α - cts soft) if $f_{pu}^{-1}(G, B) \in \alpha OS(X)$ for all $(G, B) \in OS(Y)$.*

3. *The function f_{pu} is called a semi - soft continuous function (briefly, semi - cts soft) if $f_{pu}^{-1}(G, B) \in SOS(X)$ for all $(G, B) \in OS(Y)$.*

4. *The function f_{pu} is called a β - soft continuous function (briefly, β - cts soft) if $f_{pu}^{-1}(G, B) \in \beta OS(X)$ for all $(G, B) \in OS(Y)$.*

3 δ - Open and β^* - Soft Sets

In this section, we introduce the notions of δ - open soft sets and β^* - soft sets in soft topological spaces. Also, we investigate some properties of these soft sets.

Definition 3.1. *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. (F, E) is called δ - soft set if $int(cl(F, E)) \tilde{\subseteq} cl(int(F, E))$. The set of all δ - open soft sets denoted by $\delta OS(X)$. We call a soft set $(F, E) \in SS(X)_E$ is a δ - closed soft set if its complement is δ - open soft.*

Definition 3.2. *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. (F, E) is called β^* - soft set if $(F, E) = (G, E) \tilde{\cap} (H, E)$ where (G, E) soft open and $cl(int(H, E))$ $= int(H, E)$. The set of all β^* - soft sets denoted by $\beta^* S(X)$.*

Remark 3.3. *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then every soft open set is a β^* - soft set.*

But, the converse of Remark 3.3 can not be true as the following example.

Example 3.4. *Let* $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1, e_2\}$ *and*

$$\tau = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E)\},$$

where $(F_1, E), (F_2, E)$ *are soft sets over* X *defined as follows:*

$$\begin{array}{rcl} (F_1, E) & = & \{(e_1, \{h_1, h_2\}), (e_2, X)\}, \\ (F_2, E) & = & \{(e_1, \{h_3, h_4\}), (e_2, X)\}. \end{array}$$

The soft set $(G, E) = \{(e_1, \{h_1, h_2, h_3\}), (e_2, X)\}$ *is a* β^* *- soft set of* (X, τ, E), *but it is not a soft open set.*

Proposition 3.5. *Let* $(F, E), (G, E) \in SS(X)_E$. *If*

$$(F, E) \widetilde{\subseteq} (G, E) \widetilde{\subseteq} cl(F, E) \text{ and } (F, E) \in \delta OS(X),$$

then $(G, E) \in \delta OS(X)$.

Proof. Let $(F, E) \widetilde{\subseteq} (G, E) \widetilde{\subseteq} cl(F, E)$ and $(F, E) \in \delta OS(X)$. Then, since $(F, E) \in \delta OS(X)$, we have

$$int(cl(F, E)) \widetilde{\subseteq} cl(int(F, E)).$$

Since $(F, E) \widetilde{\subseteq} (G, E)$, we obtain

$$cl(int(F, E)) \widetilde{\subseteq} cl(int(G, E))$$

and

$$int(cl(F, E)) \widetilde{\subseteq} cl(int(G, E)).$$

Since $(G, E) \widetilde{\subseteq} cl(F, E)$, we have

$$cl(G, E) \widetilde{\subseteq} cl(cl(F, E)) = cl(F, E)$$

and

$$int(cl(G, E)) \widetilde{\subseteq} int(cl(F, E)).$$

Therefore, we obtain $int(cl(G, E)) \widetilde{\subseteq} cl(int(G, E))$. This shows that $(G, E) \in \delta OS(X)$. □

Definition 3.6. *Let* (X, τ, E) *be a soft topological space and* $(F, E) \in SS(X)_E$. (F, E) *is called* τ *- soft dense if* $\widetilde{X} = cl(F, E)$.

Corollary 3.7. *Let* (X, τ, E) *be a soft topological space and* $(F, E) \in SS(X)_E$. *If* (F, E) *is* δ *- open soft and* τ*- soft dense, then every soft set over* X *containing* (F, E) *is a* δ *- open soft set.*

Proof. It is obvious. □

Proposition 3.8. *Let $(F,E),(G,E)$ and $(H,E) \in SS(X)_E$. If (F,E) is a δ - open soft set, then $(F,E) = (G,E)\widetilde{\cup}(H,E)$, where $(G,E) \in \alpha OS(X)$, $int(cl(H,E)) = \widetilde{\emptyset}$ and $(G,E)\widetilde{\cap}(H,E) = \widetilde{\emptyset}$.*

Proof. Suppose that $(F,E) \in \delta OS(X)$. Then we have

$$int(cl(F,E))\widetilde{\subseteq}cl(int(F,E))$$

and

$$int(cl(F,E))\widetilde{\subseteq}int(cl(int(F,E))).$$

Now we have

$$(F,E) = [int(cl(F,E))\widetilde{\cap}(F,E)]\widetilde{\cup}[(F,E) - int(cl(F,E)))].$$

Now, we set

$$(G,E) = int(cl(F,E))\widetilde{\cap}(F,E)$$

and

$$(H,E) = (F,E) - int(cl(F,E)).$$

We first show that $(G,E) \in \alpha OS(X)$. Now we have

$$\begin{aligned} int(cl(int(G,E))) &= int(cl(int[int(cl(F,E))\widetilde{\cap}(F,E)])) \\ &= int(cl[int(cl(F,E))\widetilde{\subseteq}int(F,E)]) \\ &= int(cl(int(F,E))). \end{aligned}$$

Since $(F,E) \in \delta OS(X)$,

$$int(cl(int(F,E)))\widetilde{\subseteq}int(cl(F,E))\widetilde{\subseteq}(G,E)$$

and thus $(G,E) \in \alpha OS(X)$. Then we show $int(cl(H,E)) = \widetilde{\emptyset}$. We obtain

$$\begin{aligned} int(cl(H,E)) &= int(cl[(F,E)\widetilde{\cap}(\widetilde{X} - int((cl(F,E)))]) \\ &\widetilde{\subseteq}int(cl(F,E))\widetilde{\cap}int(cl(\widetilde{X} - int(cl(F,E)))) \\ &\widetilde{\subseteq}int(cl(F,E))\widetilde{\cap}(\widetilde{X} - int(cl(F,E))) \\ &= \widetilde{\emptyset}. \end{aligned}$$

It is obvious that

$$(G,E)\widetilde{\cap}(H,E) = [int(cl(F,E))\widetilde{\cap}(F,E)]\widetilde{\cap}[(F,E) - int(cl(F,E))] = \widetilde{\emptyset}.$$

□

Theorem 3.9. *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. (F, E) is a semi - open soft if and only if it is both δ - open soft and β - open soft set.*

Proof. Let (F, E) be a semi - open soft set, then we have

$$(F, E)\widetilde{\subseteq}cl(int(F, E))\widetilde{\subseteq}cl(int(cl(F, E))).$$

This shows that (F, E) is a β - open soft set. Moreover,

$$int(cl(F, E))\widetilde{\subseteq}cl(F, E)\widetilde{\subseteq}cl(cl(int(F, E))) = cl(int(F, E)).$$

Therefore, (F, E) is a δ - open soft set.

Conversely, let (F, E) be a δ - open soft and β - open soft set, then we have

$$int(cl(F, E))\widetilde{\subseteq}cl(int(F, E)).$$

Thus we obtain that

$$cl(int(cl((F, E)))\widetilde{\subseteq}cl(cl(int(F, E))) = cl(int(F, E)).$$

Since (F, E) is β - open soft, we have

$$(F, E)\widetilde{\subseteq}cl(int(cl(F, E)))\widetilde{\subseteq}cl(int(F, E))$$

and

$$(F, E)\widetilde{\subseteq}cl(int(F, E)).$$

Hence (F, E) is a semi - open soft set. □

Theorem 3.10. *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. (F, E) is an α - open soft set if and only if it both δ - open soft and pre - open soft.*

Proof. Let (F, E) be an α - open soft set. Since every α - open soft set is a semi - open soft set, by Theorem 3.9. (F, E) is a δ - open soft set. Also, every α - open soft set is pre - open soft.

Conversely, let (F, E) be both δ - open soft and pre - open soft set. Then we have

$$int(cl(F, E))\widetilde{\subseteq}cl(int(F, E))$$

and so

$$int(cl(F, E))\widetilde{\subseteq}int(cl(int(F, E))).$$

Since (F, E) is pre - open soft, we have $(F, E)\widetilde{\subseteq}$ $int(cl(F, E))$. Therefore we obtain $(F, E)\widetilde{\subseteq}int(cl(int(F, E)))$ and hence (F, E) is an α - open soft set. □

Theorem 3.11. *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is a soft open set if and only if it is both α - open soft set and β^* - soft set.*

Proof. Let $(F,E) \in \tau$, so $(F,E) = (F,E)\widetilde{\cap}(\widetilde{X},E)$ where $(F,E) \in \tau$ and $cl(int(\widetilde{X})) = int(\widetilde{X}) = \widetilde{X}$. Hence (F,E) is a β^* - soft set. Clearly (F,E) is an α - open soft set.

Conversely, let (F,E) be an α - open soft set and β^* - soft set. So we get

$$(F,E)\widetilde{\subseteq}int(cl(int(F,E)))$$

and

$$(F,E) = (G,E)\widetilde{\cap}(H,E),$$

where (G,E) soft open set and $cl(int(H,E)) = int(H,E)$. So, we obtain

$$\begin{aligned}
(F,E)\widetilde{\subseteq}int(cl(int(F,E))) &= int(cl(int((G,E)\widetilde{\cap}(H,E))))\\
&= int(cl(int(G,E)\widetilde{\cap}int(H,E))))\\
&= int(cl((G,E)\widetilde{\cap}int(H,E)))\\
&\widetilde{\subseteq}int(cl(G,E)\widetilde{\cap}cl(int(H,E)))\\
&= int(cl(G,E)\widetilde{\cap}int(H,E)))\\
&= int(cl(G,E))\widetilde{\cap}int(H,E).
\end{aligned}$$

Hence $(F,E)\widetilde{\subseteq}(G,E)\widetilde{\cap}int(H,E)$, we have (F,E) soft open set. □

Remark 3.12. *By the following examples stated below, we obtain the following results:*

1. *δ - open soft set and β - open soft set are not required each other.*
2. *δ - open soft set and pre - open soft set are not required each other.*
3. *β^* - soft set and α - open soft set are not required each other.*

Example 3.13. *Let $X = \{a,b,c,d\}$, $E = \{e_1,e_2\}$ and*

$$\tau = \{\widetilde{X}, \widetilde{\emptyset}, (F_1,E), (F_2,E)\},$$

where $(F_1,E),(F_2,E)$ are soft sets over X defined as follows:

$$\begin{aligned}
(F_1,E) &= \{(e_1,\{d\}),(e_2,\{a,b,c\})\},\\
(F_2,E) &= \{(e_1,\{a,b,c\}),(e_2,\{d\})\}.
\end{aligned}$$

The soft set $(G,E) = \{(e_1,\{c,d\}),(e_2,\{c,d\})\}$ is a pre - open soft set of (X,τ,E), but it is not a δ - open soft set.

Example 3.14. *Let $X = \{a,b,c,d\}$, $E = \{e_1,e_2\}$ and*

$$\tau = \{\widetilde{X}, \widetilde{\emptyset}, (F_1,E), (F_2,E)\},$$

where $(F_1, E), (F_2, E)$ are soft sets over X defined as follows:

$$\begin{aligned} (F_1, E) &= \{(e_1, \{a\}), (e_2, \{a, b, c\})\}, \\ (F_2, E) &= \{(e_1, \{a, b, c\}), (e_2, \{a\})\}. \end{aligned}$$

The soft set $(G, E) = \{(e_1, \{b, c\}), (e_2, X)\}$ is a δ - open soft set of (X, τ, E), but it is not a β - open soft set.

Example 3.15. *Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1, e_2\}$ and*

$$\tau = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\},$$

where $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ are soft sets over X defined as follows:

$$\begin{aligned} (F_1, E) &= \{(e_1, \{h_1, h_3\}), (e_2, \{h_2, h_4\})\}, \\ (F_2, E) &= \{(e_1, \{h_2, h_4\}), (e_2, \{h_1, h_3\})\}, \\ (F_3, E) &= \{(e_1, \{h_1\}), (e_2, \{h_1\})\}, \\ (F_4, E) &= \{(e_1, \{h_1, h_2, h_4\}), (e_2, \{h_1, h_2, h_4\})\}. \end{aligned}$$

The soft set $(G, E) = \{(e_1, \{h_2, h_3, h_4\}), (e_2, \{h_1, h_3, h_4\})\}$ is a β^* - soft set of (X, τ, E), but it is not an α - open soft set.

Example 3.16. *Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and*

$$\tau = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E)\},$$

where (F_1, E) are soft sets over X defined as follows:

$$(F_1, E) = \{(e_1, \{h_2\}), (e_2, \{h_3\})\}.$$

The soft set $(G, E) = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_3\})\}$ is an α - open soft set of (X, τ, E), but it is not a β^* - soft set.

Remark 3.17. *We obtain the following diagram.*

DIAGRAM - I

$$\begin{tikzcd}
\beta^* S(X) & OS(X) \arrow[l] \arrow[r] \arrow[d] & \alpha OS(X) \arrow[r] \arrow[dl] & SOS(X) \arrow[r] \arrow[dl] & \delta OS(X) \\
& POS(X) \arrow[r] & \beta OS(X) & &
\end{tikzcd}$$

4 Decompositions in Soft Topological Spaces

In this section, we obtain some new decompositions of soft continuity, semi - soft continuity and α - soft continuity.

Theorem 4.1. *[3] Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then*

1. *every soft continuous function is a pre - soft continuous function,*
2. *every soft continuous function is a semi - soft continuous function,*
3. *every soft continuous function is an α - soft continuous function,*
4. *every soft continuous function is a β - soft continuous function.*

Theorem 4.2. *[3] Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then*

1. *every α - soft continuous function is a semi - soft continuous function,*
2. *every semi - soft continuous function is an β - soft continuous function,*
3. *every pre - soft continuous function is an β - soft continuous function,*
4. *every α - soft continuous function is a pre - soft continuous function.*

Theorem 4.3. *[3] Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then f_{pu} is an α - soft continuous function if and only if it is a pre - soft continuous and semi - soft continuous function.*

Definition 4.4. *Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. The function f_{pu} is called δ - soft continuous (briefly, δ - cts soft) if $f_{pu}^{-1}(G, B) \in \delta OS(X)$ for all $(G, B) \in \tau^*$*

Definition 4.5. *Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then The function f_{pu} is called β^* - soft continuous (briefly, β^* - cts soft) if $f_{pu}^{-1}(G, B) \in \beta^* S(X)$ for all $(G, B) \in \tau^*$.*

Theorem 4.6. *Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then the following are equivalent:*

1. *f_{pu} is semi - soft continuous;*

2. f_{pu} *is both* β *- soft continuous and* δ *- soft continuous.*

Proof. It is clear from Theorem 3.9. □

Theorem 4.7. *Let* (X, τ, A) *and* (Y, τ^*, B) *be soft topological spaces. Let* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. Then the following are equivalent:*

1. f_{pu} *is* α *- soft continuous;*
2. f_{pu} *is both pre - soft continuous and semi - soft continuous;*
3. f_{pu} *is both pre - soft continuous and* δ *- soft continuous.*

Proof. It is obvious from Theorems 3.10 and 4.3. □

Theorem 4.8. *Let* (X, τ, A) *and* (Y, τ^*, B) *be soft topological spaces. Let* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. Then the following are equivalent:*

1. f_{pu} *is soft continuous;*
2. f_{pu} *is both* α *- soft continuous and* β^* *- soft continuous.*

Proof. It is obvious from Theorem 3.11. □

Remark 4.9. *By the following examples stated below, we obtain the following results:*

1. β *- soft continuous function and* δ *- soft continuous function are not required each other.*
2. *pre - soft continuous function and* δ *- soft continuous function are not required each other.*
3. β^* *- soft continuous function and* α *- soft continuous function are not required each other.*

Example 4.10. *Let* $X = Y = \{a, b, c, d\}$ *and* $A = B = \{e_1, e_2\}$*. Let* $u : X \to Y$ *and* $p : A \to B$ *be defined by identity function. Also, let*

$$\tau = \{\widetilde{\emptyset}, \widetilde{X}, (F_1, A), (F_2, A)\}$$

and

$$\tau^* = \{\widetilde{\emptyset}, \widetilde{X}, (G, B)\},$$

where $(F_1, A), (F_2, A), (G, B)$ *are soft sets over* X *defined as follows:*

$$\begin{aligned}(F_1, A) &= \{(e_1, \{a, c\}), (e_2, \{a, b, c\})\},\\ (F_2, A) &= \{(e_1, \{a, b, c\}), (e_2, \{a, c\})\},\\ (G, B) &= \{(e_1, \{a, b\}), (e_2, Y)\}.\end{aligned}$$

Then f_{pu} *is a pre - soft continuous function, but it is not* δ *- soft continuous.*

Example 4.11. *Let* $X = Y = \{a, b, c, d\}$ *and* $A = B = \{e_1, e_2\}$*. Let* $u : X \to Y$ *and* $p : A \to B$ *be defined by identity function. Also, let*

$$\tau = \{\widetilde{\emptyset}, \widetilde{X}, (F_1, A), (F_2, A)\}$$

and

$$\tau^* = \{\widetilde{\emptyset}, \widetilde{X}, (G, B)\},$$

where $(F_1, A), (F_2, A), (G, B)$ *are soft sets over* X *defined as follows:*

$$\begin{aligned}(F_1, A) &= \{(e_1, \{b\}), (e_2, \{a, b, c\})\},\\ (F_2, A) &= \{(e_1, \{a, b, c\}), (e_2, \{b\})\},\\ (G, B) &= \{(e_1, \{a, c\}), (e_2, Y)\}.\end{aligned}$$

Then f_{pu} *is a* δ *- soft continuous function, but it is not* β *- soft continuous.*

Example 4.12. *Let* $X = \{x, y, z\}$*,* $Y = \{x, y\}$ *and* $A = B = \{e_1, e_2\}$*. Let* $u : X \to Y$ *be defined by* $u(x) = u(y) = x, u(z) = y$ *and* $p : A \to B$ *be defined by* $p(e_1) = e_1$*. Also, let*

$$\tau = \{\widetilde{\emptyset}, \widetilde{X}, (F, A)\}$$

and

$$\tau^* = \{\widetilde{\emptyset}, \widetilde{X}, (G, B)\},$$

where $(F, A), (G, B)$ *are soft sets over* X *defined as follows:*

$$(F, A) = \{(e_1, \{x\}), (e_2, \{x\})\}$$

and

$$(G, B) = \{(e_1, \{x\}), (e_2, Y)\}.$$

Then f_{pu} *is an* α *- soft continuous function, but it is not* β^* *- soft continuous.*

Example 4.13. *Let* $X = \{x, y, z, t\}$, $Y = \{x, y, z\}$ *and* $A = B = \{e_1, e_2\}$. *Let* $u : X \to Y$ *be defined by* $u(x) = u(y) = x, u(z) = y, u(t) = z$ *and* $p : A \to B$ *be defined by* $p(e_1) = e_1$. *Also, let*

$$\tau = \{\widetilde{\emptyset}, \widetilde{X}, (F_1, A), (F_2, A)\}$$

and

$$\tau^* = \{\widetilde{\emptyset}, \widetilde{X}, (G, B)\},$$

where $(F_1, A), (F_2, A), (G, B)$ *are soft sets over* X *defined as follows:*

$$\begin{aligned} (F_1, A) &= \{(e_1, \{x, y\}), (e_2, \{z, t\})\}, \\ (F_2, A) &= \{(e_1, \{z, t\}), (e_2, \{x, y\})\} \end{aligned}$$

and

$$(G, B) = \{(e_1, \{x, y\}), (e_2, \{x, y\})\}.$$

Then f_{pu} *is a* β^* *- soft continuous function, but it is not* α *- soft continuous.*

Remark 4.14. *We obtain the following diagram which shows relationships among* β^* *- soft continuity,* δ *- soft continuity and other soft continuities.*

DIAGRAM - II

$$\begin{tikzcd}
\beta^*\text{-soft cts} & \text{soft cts} \arrow[l] \arrow[rr] \arrow[dr] & & \alpha\text{-soft cts} \arrow[rr] \arrow[dl] & & \text{semi-soft cts} \arrow[r] \arrow[dl] & \delta\text{ - soft cts} \\
& & \text{pre-soft cts} \arrow[rr] & & \beta\text{-soft cts} & &
\end{tikzcd}$$

Definition 4.15. *Let* (X, τ, A) *and* (Y, τ^*, B) *be soft topological spaces and* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. Then* f_{pu} *is called*

1. α *- open soft if* $f_{pu}(G, A) \in \alpha OS(Y)$ *for all* $(G, A) \in \alpha OS(X)$,
2. β *- open soft if* $f_{pu}(G, A) \in \beta OS(Y)$ *for all* $(G, A) \in \beta OS(X)$,
3. *semi - open soft if* $f_{pu}(G, A) \in SOS(Y)$ *for all* $(G, A) \in SOS(X)$,
4. *pre - open soft if* $f_{pu}(G, A) \in POS(Y)$ *for all* $(G, A) \in POS(X)$,
5. δ *- open soft if* $f_{pu}(G, A) \in \delta OS(Y)$ *for all* $(G, A) \in \delta OS(X)$,
6. β^* *- open soft if* $f_{pu}(G, A) \in \beta^* S(Y)$ *for all* $(G, A) \in \beta^* S(X)$.

Theorem 4.16. *Let (X, τ, A) and (Y, τ^*, B) be soft topological spaces. Put the functions $u : X \to Y$, $p : A \to B$ and $f_{pu} : SS(X)_A \to SS(Y)_B$. Then the followings hold:*

1. *f_{pu} is α - open soft if and only if it is both pre - open soft and δ - open soft,*
2. *f_{pu} is open soft if and only if it is both α - open soft and β^* - open soft,*
3. *f_{pu} is semi - open soft if and only if it is both β - open soft and δ - open soft.*

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